Transmission Lines, Pads and Attenuators

LESSON ND-30

SPRAYBERRY ACADEMY of RADIO

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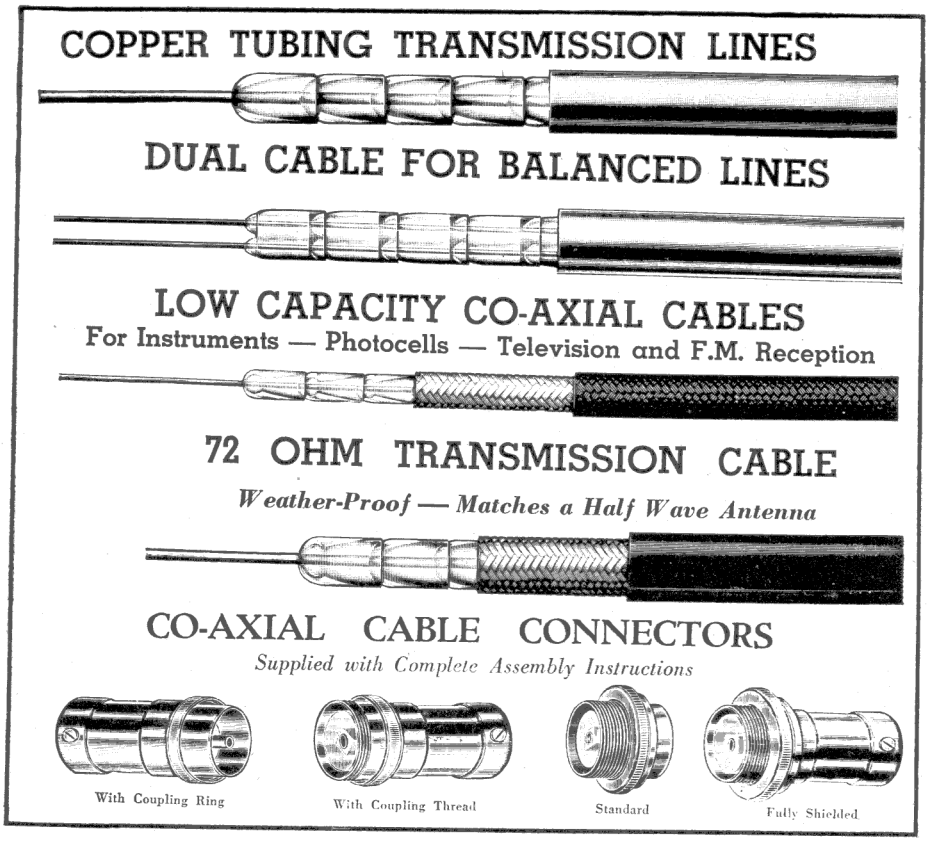
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With Coupling Ring
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Standard
Fully Shielded

The above view shows several types of transmission lines as used in high frequency transmission. Practical applications are to be found in television, frequency modulation, transmitters and antenna circuits. Note that screw type connectors are used at the end of the cables.

TRANSMISSION LINES, PADS AND ATTENUATORS

LESSON ND-30

This lesson deals with complex subjects which cannot be adequately explained except by involved mathematics. The sudent should at least have a general knowledge of these subjects because at various times he will find it necessary to work with transmission lines, pads and attenuators. The subjects assume more importance if the student expects to work in radio stations, public address systems, motion pictures, television, aircraft or on board ship. If, on the other hand, the student expects most of his endeavors in radio to be confined to service and repair work, then he need not be too greatly concerned with the details of this lesson. All radio men, however, should have a general knowledge of the subjects in this lesson, and

you are urged to give it careful study.

A signal transmission line consists of two or more conductors. It may be one wire with an outside metal shield—the shield acting as one conductor. Other forms are two parallel wires, two twisted wires, two transposed wires, etc.

Transmission lines are used in radio for many purposes. For instance, in public address work, in the broadcast studios and at remote signal pick-up spots, one or more microphones must be coupled or connected to the amplifying equipment. A cable (transmission line) of various lengths must be used for this purpose.

For other work, it may be required to couple a phonograph pick-up unit to amplifying equipment—a transmission line must be used for this purpose. A transmitter requires a transmission line to feed its energy to an antenna and the receiver requires a transmission line to feed the picked up signal at the antenna to the receiver. Signals must also be fed to telephone lines. In all of these and other applications, connecting wires or cables are required between units—thus, transmission lines are highly important to all who are engaged in any technical radio work.

Signal strengths along these lines must also be controlled. In one place, it may be desired to *decrease* the signal strength. In another place it may be desired to *increase* the signal strength. Likewise, it is often necessary to *mix* one or more signals in a common circuit—that is, fade in one or fade out another. This is all accomplished by making proper use of *pads and attenuators* (essentially volume control units). These must be connected to proper transmission lines and to impedance matching transformers. So, from this brief description you can see that all of these various factors are all closely related, and at least a working or general knowledge of them is required in practically all radio work.

In studying this lesson, do not let the mathematics or the more or less intangible theory of the subjects discourage you. Remember, as a practical working man, you will not be expected to be thoroughly familiar with design data associated with this work. However, it is necessary to go into the subject in detail in order to give you reasons and proof of why certain facts exist. So please bear this in mind as you read the following pages. You will be shown why certain things are true, and from this you will know how to handle manufactured devices which have been already designed for you.

In the main, these subjects involve the transfer of electrical impulses or signals over various distances, the control of the level of the strength of the signal and *the correction of variations imposed on the signal by the transmission circuit or system.* First the transmission lines which are used to carry radio or audio signals from one point to another will be described.

The most common type of line consists of two parallel copper wire conductors, separated only by their insulation. These may be twisted or bound together in a common sheath. An average size

wire may be considered to make up the conductor—the wire may be solid or stranded.

The proximity of the two wires and the fact that they are parallel to each other naturally means that they must have a certain amount of capacity between them. Or, stating it another way—the two wires form a capacity. The wires have a certain surface area facing each other and are separated by a dielectric. The line is, of course, considered to be uniform and from this you may assume that the capacity between wires is exactly proportional to the length of the line. If the length of the line is tripled for example, the total capacity formed by the two wires is tripled.

Since the capacity of the line is in shunt with the signal which is traveling over the line, it will tend to by-pass the signal rather than to pass it on to the other end of the line as would a coupling condenser. Moreover, this by-passing effect will be increased with the length of the line due to the increased capacity encountered. Knowing the relation of capacity and reactance, you can see at once that the high frequencies will tend to lose more energy across the line than the low frequencies because of the lower reactance and, further, due to the capacity increasing directly with length, the low frequencies will be attenuated somewhat less than the high frequencies. Or, if you care to look at it in another way, for the same amount of loss the lower the frequency, the further the signal can travel along the line.

Keep the foregoing facts in mind when considering the following. Since the line is composed of wire, naturally you would expect that a sufficient length of it will form a definite value of inductance. That is, any current flow through the wire will form a magnetic field around the wire which will influence the applied current of the line. This, you will remember, is how inductance is formed.

This inductance is in series with the line, and its reactance increases with an increase in frequency. The transmission line will, therefore, tend to retard the higher frequencies or cut them down more rapidly than it will affect the lower frequencies. At first, thought, it my seem that these two effects (shunt capacity and series inductance) will cancel each other and that all frequencies will experience the same amount of loss in a given length of line. Unfortunately, it is not nearly so simple as that.

One of the first experiments with transmission lines to find out just how they would affect a signal was to connect two ends of a line to an *impedance bridge* supplied with a variable frequency signal. Assume, for example, that this line is 300 feet long and open at the far end. The impedance bridge will indicate that at certain frequencies not harmonically related or numerically related *the impedance of the line will be very low,* while at other frequencies or for complete bands, *the impedance will be high.* In some sections of the audio spectrum, the transmission line will appear to have regular resonance peaks, just as though it were a series resonant circuit. However, this cannot be true because of the

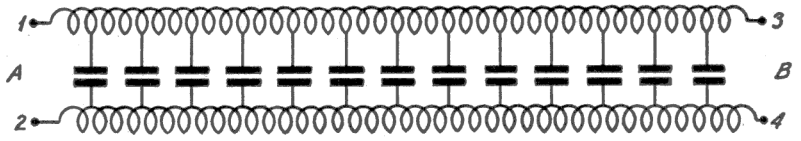


FIG. 1

other similar actions the transmission line exhibits as though replaced by other resonant circuits. No system of circuits can be constructed exactly equivalent to the transmission line and because of this the transmission line must be treated as a separate subject.

The transmission line may be symbolized in its true electrical form as in Fig. 1. The inductance along the line is symbolized as a coil while the capacity may be symbolized as a number of separate condensers. Just assume that there are a *very large number of small condensers* across the line as shown. The input is regarded as points 1 and 2 at A, while the output is at B and is represented by points 3 and 4.

To arrive at a quantitative value for the line, its impedance must be expressed in some form which will be *a true representation of the line.* With capacity there is leakage across the line; the two components of which combine to form impedance rather than pure reactance. Therefore, *the impedance across the line* can be represented

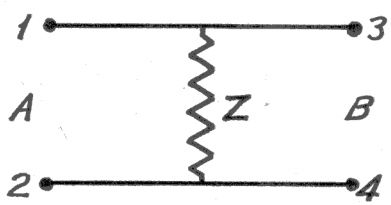


FIG. 2

as a single impedance and *symbolized as a resistor* as shown by Z in Fig. 2. In an electrical sense, the *series impedance* (formed by the inductive reactance and DC resistance) may be represented by a single impedance also symbolized as a resistance Z. This is shown in Fig. 3, with each *series impedance* split in the center where the *shunt impedance* is attached. Mathematically, you may disregard the impedance in one side of the line and consider it all in the other side. The final equivalent circuit for the purpose of this discussion is shown in Fig. 4.

There is no information of the relative magnitude of the series

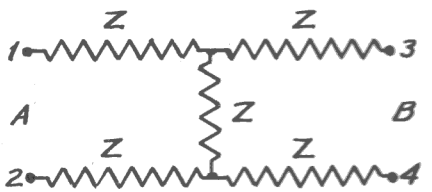


FIG. 3

and shunt impedances, so they are simply identified as Z-1 (series) and Z-2 (shunt) as in Fig. 5. With a little thought it is obvious that if you measure with a suitable instrument from terminals 1 and 2 in Fig. 5, you will not get the same value as between terminals 3 and 4. In the first case, you will measure the sum of Z-1 and Z-2, and, in the second case, you will measure only the value of Z-2. From a study of Fig. 1, you will note there is no

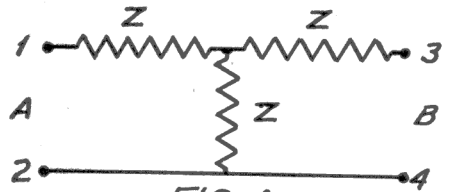


FIG. 4

reason to believe that terminals 1 and 2 would test any differently from 3 and 4 because of the uniformity and symmetry of the lines. Therefore, Z-1 may be divided in two, placing Z-2 in shunt between each half of Z-1 as shown in Fig. 6. This provides the proper electrical equivalent circuit of the transmission line.

This circuit of Fig. 6 *is a fundamental transmission line circuit and also an electrical network equivalent circuit.* While some electrical networks are enormously difficult to analyze and to convert to simpler forms for exact study or measurement, they all may be reduced to the form of Fig. 6.

This simplification of circuits to equivalents is the only practical way known to get tangible information about such a circuit as the transmission line. It provides a way to find how the circuit, no matter how complicated, acts on signals of any frequency passed through them. This three section circuit of Fig. 6, the elements of which form a letter *T*, is commonly called a *T* circuit, or, in some

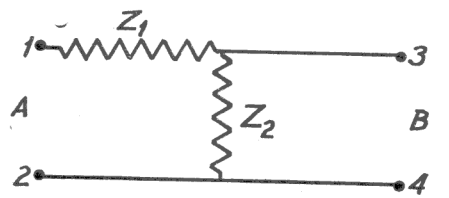


FIG. 5

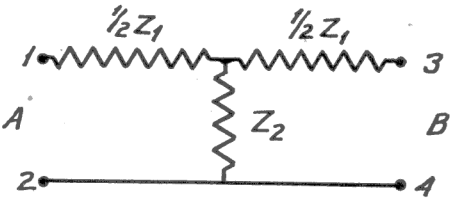


FIG. 6

cases, a *Star* circuit. It is as basic to the study of electrical networks and transmission lines as Ohm's law is to electricity.

To simplify the discussion, the signal may be limited to *one frequency* and with a signal of this frequency feeding terminals 1 and 2, Fig. 7, assume that the applied load ZL is varied through every possible value—that is, from zero to infinity or from a direct short to an open. For this, you will be interested in the impedance measured across terminals 1 and 2 (the input terminals) for various values of load or output impedance ZL. It will be remembered that the nature of the impedance of the line is the only thing that can affect the signal's passage through it. This includes its resistance, leakage, etc., as explained.

With ZL in Fig. 7 shorted, the circuit reduces to a simple series parallel group of impedances consisting of ½ Z-1 in series with the parallel group of ½ Z-1 and Z-2. The total impedance between terminals 1 and 2 would, therefore, be equal to:

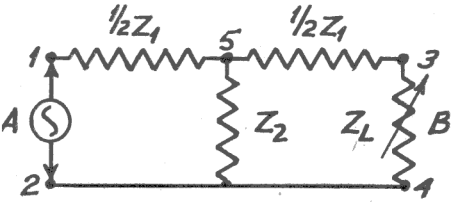


FIG. 7

$$\tfrac{1}{2} Z_1 + \frac{\tfrac{1}{2} Z_1 Z_2}{\tfrac{1}{2} Z_1 + Z_2}$$

the second part of which may be simplified to obtain a total of—

$$\tfrac{1}{2} Z_1 + \frac{Z_1 Z_2}{Z_1 + 2Z_2} \qquad (1)$$

If the load is open or infinte, the impedance of the input will simply be the sum of—

$$\tfrac{1}{2} Z_1 + Z_2 \qquad (2)$$

From your former studies, you have learned that the effective value of two resistors in parallel cannot possibly be as high as either of the single value. In the same sense, formula (1) includes two impedances in parallel. The value of the entire expression can never be as high as formula (2) in which no impedance is across Z_2.

With any other value of ZL across the output of the line, the impedance at the input A will have a value somewhere between the two values determined by formulas 1 and 2.

If you include ZL in the circuit having some value other than zero or infinite, you may write the impedance at the input Zs (*source impedance*) as follows:

$$Zs = \frac{Z_1}{2} + \frac{Z_2 \left(\frac{Z_1}{2} + ZL \right)}{Z_2 + \frac{Z_1}{2} + ZL}$$

which simplifies as follows:

$$Zs = \frac{Z_1}{2} + \frac{\frac{Z_1 Z_2}{2} + Z_2 ZL}{Z_2 + \frac{Z_1}{2} + ZL}$$

$$= \frac{Z_1}{2} + \frac{Z_1 Z_2 + 2Z_2 ZL}{2Z_2 + Z_1 + 2ZL} \qquad (3)$$

Values for Z_1 and Z_2 may now be assumed which are logically within the range in which you are studying and values may be plotted for the input impedance for various values of the load impedance ZL. With this method of study, higher mathematics are avoided and each solution is obtained with a separate problem.

Assume Z_1 to be 200 ohms and Z_2 to be 100 ohms for the entire discussion. Then assume ZL to be 10 ohms for the first example. Substituting these values in formula 3 and working out the problem, you will obtain the following:

$$Zs = \frac{200}{2} + \frac{200 \times 1000 + 2(1000\ ZL)}{2 \times 1000 \times 200 + 2ZL}$$

$$Zs = 100 + \frac{200{,}000 + 2000ZL}{2200 + 2ZL} \qquad (3A)$$

Now with ZL equal to 10 you will obtain:

$$Zs = 100 + \frac{200{,}000 + 2000 \times 10}{2200 + 2 \times 10}$$

$$= 100 + \frac{220{,}000}{2220}$$

$$= 100 + 99.099$$

$$= 199.099 \text{ or } 199 \text{ ohms.}$$

Thus, for other values of ZL substituted in equation (3A) the following values for Zs are obtained.

ZL	Zs
10	199
100	266.8
500	475
1,000	623.5
10,000	1010
100,000	1091

Although rapidly rising values of ZL have been chosen in the foregoing, it will be noted that Zs does not rise so rapidly as a result. In fact, from inspection of Fig. 7, you will note that even if the load is removed the total value of the input impedance Zs cannot possibly exceed 1100 ohms, as this value

is formed by one section of Z_1 and Z_2. The values of ZL chosen start somewhat below the Zs values resulting, but somewhere between 100 and 500 ohms for ZL—the two must be equal as the ZL values above this point exceed the Zs values.

Now for the present considering the intermediary values of impedance between the source (Zs) and the load (ZL), you will see an immediate advantage in the source impedance value equalling the load impedance value. *When this condition is obtained, the transfer of power is maximum in a transmission line.*

You will, no doubt, be interested in knowing what values the source and load impedances are when they are equal and just why this is true. You may also like to know what makes them require this value. If, in the final analysis, ZL and Zs are to be equal, you may simply substitute ZL in place of Zs in equation (3), obtaining:

$$ZL=100+\frac{200{,}000+2000ZL}{2200+2ZL}=$$

$$\frac{220{,}000+200ZL+200{,}000+2000ZL}{2200+2ZL}$$

(Combining terms)

$$2200ZL=2ZL^2$$

$$=220000+200ZL+200000+2000ZL$$

(Clearing fractions)

$$2ZL^2=200ZL-2200ZL+2000ZL+$$

$$220{,}000+200{,}000 \text{ (cancellation)}$$

$$2ZL^2=420{,}000$$

$$ZL^2=210{,}000$$

$$ZL=\sqrt{210{,}000}$$

$$=458 \text{ ohms.}$$

This simply means that if a value of 458 ohms is placed across the output of Fig. 7 as ZL, *the impedance that you would measure*

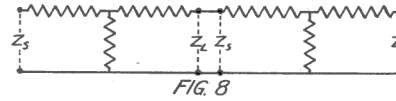


FIG. 8

across the input would be 458 ohms also. A graph of this information or further conclusive mathematics would prove this is *the only value* for ZL which would result in the same value for Zs.

Suppose, as in Fig. 8, another identical *T* section is used for the load as 1100 ohms, which value it will have, as mentioned previously, provided its end or output is not loaded. Now with 1100 ohms loading the first *T* section, its source impedance will be slightly greater than 623.5 ohms as shown in the previous table for the 1000 ohm load. If you place still another section to the right of this one and connect it, its source impedance will drop a little more than 623.5 ohms, and this value will be the load for the first unit. *With this additional load, the input impedance of the first T section* will be still lower. Finally, if you add more sections as per Fig. 9, the first source impedance Zs will approach 458 ohms. If, at the end of *any T* section, you connect a 458 ohm load the source impedance will measure 458 ohms also. Theoretically, if you add an infinite number of sections, you can leave the end section open and still the source impedance will remain at 458 ohms. In practice, about 7 or 8 such sections would bring the source impedance to within ½ or 1 percent of this figure (458 ohms). Remember in all of this discussion the word *ohm* refers to AC impedance and not DC resistance.

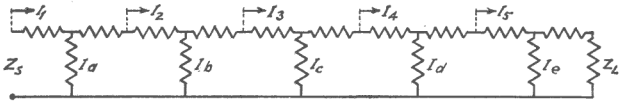


FIG. 9

This value of impedance—that is, the number of ohms which, if loaded by this amount, will make the source matched to it, is *characteristic* of this particular type of line or *T* section. It is true for no other section unless electrically like this one. It is, therefore, referred to as the *characteristic impedance of the transmission line.*

When you are dealing with multiple *T* sections of a transmission line you must take one other factor into account. The signal voltage will create an initial current (I_1) in the series section of the first branch, part of which will be passed on to the other branches as I_2. Some of the initial current, I_1, will be lost across the shunt section indicated in Fig. 9 as Ia. A portion of the entire signal current will be lost across each shunt section as Ib, Ic, Id, etc., and this will, of course, determine the amount of current at the ultimate output. However, if you can establish any relation between I_1 and I_2 and the other signal currents in the series branches, you can determine the reduction of current for any number of T sections. It will be noticed that the series elements are essentially small as compared to the shunt elements for the example chosen, and for that reason the current lost per section will be low as compared to that transmitted for equal sections as mentioned in connection with Fig. 9. The current entering the first section will naturally be larger than that entering the second, and the current entering the third section will be smaller than that entering the second, etc. However, the first section will reduce the current by a given *percentage* while the second section will reduce it by the same *percentage* because the ratio of series to shunt values of impedance are the same for all sections.

The ratio of this first current I_1 to the second current I_2 is—

$$\frac{I_1}{I_2} = \frac{\frac{Z_1}{2} + Z_2 + Zs}{Z_2}$$

and substituting values in the equation you will obtain:

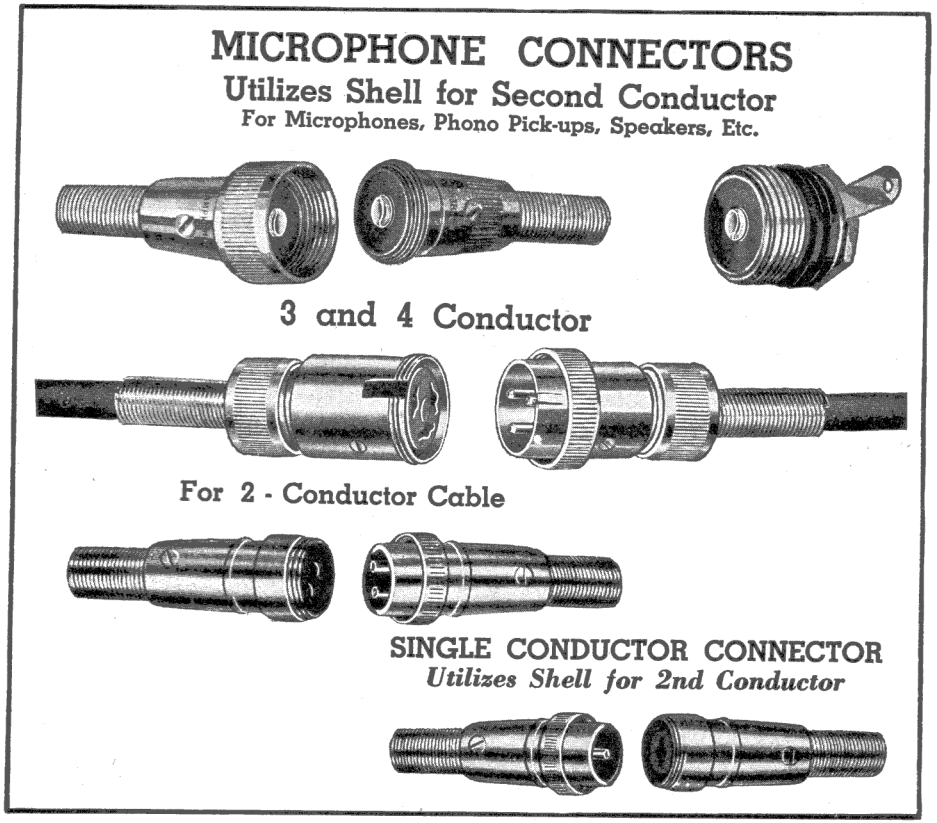
$$\frac{I_1}{I_2} = \frac{100+1000+458}{1000}$$

$$\frac{I_1}{I_2} = \frac{1558}{1000} = 1.558$$

This, of course, applies when the first section is loaded with its characteristic impedance or this impedance is placed at the end of any sections which may be added. This represents a drop in signal current of 55.7% or 3.846 db from the relation:

$$\begin{aligned} db &= 20 \log \frac{I_1}{I_2} \\ &= 20 \log 1.55 \\ &= 20 \times .1923 \\ &= 3.846 \end{aligned}$$

This ratio of the current entering the first section to that entering the next section is known as the *attenuation constant* of the line. Each section exactly like this will attenuate the signal 3.846 db. Hence, for ten such sections, there would be a total loss of the signal of 38.46 db.


MICROPHONE CONNECTORS
Utilizes Shell for Second Conductor
For Microphones, Phono Pick-ups, Speakers, Etc.
3 and 4 Conductor
For 2 - Conductor Cable
SINGLE CONDUCTOR CONNECTOR
Utilizes Shell for 2nd Conductor

In the above view are shown several types of connectors that are used in connection with transmission lines, amplifiers, microphones, pick-up units and photo-cells. These are available for different numbers of conductors, depending upon type of equipment used and the amount of power to be transmitted.

While these ideas are not sufficient to inform you as to quantitive values of various kinds of transmission lines, they show the nature of transmission lines having what is known as *distributed constants*. The inductance is evenly distributed throughout the length of the line rather than in one or a number of places. Likewise, the leakage and reactance across the line is evenly distributed on the line. One inch or one mile would have proportional quantities of every one of these characteristics.

Considering the RMS values of the signal for each point in the line, a graph of the logarthmic type is shown in Fig. 10, indicating just how the signal is reduced along the line. All attenuation factors or degrees cannot be shown in Fig. 10 as this would require three dimensions, but a curve is shown for several amounts of attenuation. It will be noted that for equal distances of the line in arbitrary units, successive values of the signal have a constant percentage relationship.

For a long line, just as for space, the instantaneous values of the signal are different for different points along the line. A sine wave voltage introduced at the line input

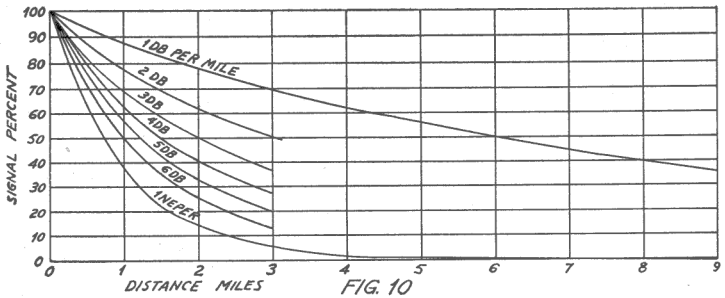


FIG. 10

at peak value will not only have a reduced value because of the logarithmic attenuation but also because of its cycle change in time. As the peak voltage travels down the line, it will be followed by lesser voltage values for $\frac{1}{4}$ of a cycle at which time the input voltage will be zero. The exact voltage at any point for a particular time is, therefore, not only a matter of attenuation but also the *phase* of the voltage in its change throughout the cycle. Now any series inductance will have the effect of reducing the velocity of the charge representing the signal voltage as it travels over the line. The shunt capacity will also reduce the velocity of the charge. While in space or vacuum a wave has a velocity of approximately 186,000 miles per second, because of their electrical characteristics, transmission lines all produce lower rates of speed.

In general, for open wire lines where the inductance is small and the shunt capacity is very small, the propagation rate is high—within a few percent of that in space. Obviously, then, the charge representing, for example, the peak signal reaches the load before its phase changes much at the source. The changes in phase of the source and load potentials are almost simultaneous. Making use of the familiar formula for velocity of a radio wave in space, for example, at 60 cycles, one wavelength would be 186,000/60 or 3100 miles. One quarter of a cycle would occupy a space along the line 3100/4 or 774 miles. In other words, if you were to attempt transmission of a 60 cycle signal over this distance, the voltage at the source would drop exactly to zero when it was at maximum at the load end.

However, electrical power transmission lines are very much shorter than this, and, in many cases, only 25 cycles are used for its transmission. For the 60 cycle line, the phase change along the line would be only 1 degree for every 8.6 miles, approximately. This length is such a small fraction of the wavelength that the electrical properties of the line lose significance and need not be considered. This factor for audio frequencies will be explained later. At full load current, the load impedance is simply made to match the line. Load and source impedance may be changed, and the difference in transmission efficiency will be negligible.

In communication, however, the average frequencies are considerably higher and the line specifications are much more closely controlled. Thus, if any line is short

as compared to the wavelength of the signal being transmitted over it, its electrical properties may be neglected. The load is simply matched to the source. However, it is usually necessary to deal with lines whose electrical characteristics must be considered as a part of the system.

Referring now to Fig. 6, it will be shown how the *characteristic impedance* may be found from a knowledge of the series and shunt impedances. An expression for the impedance of the source is given in equation (3). This is repeated below.

$$Zs = \frac{Z_1}{2} + \frac{Z_2\ (\frac{Z_1}{2} + ZL)}{Z_2 + \frac{Z_1}{2} + ZL}$$

As before, ZL and Zs will be considered to be equal using only the one term Zs to replace ZL. From the third step of formula 3 the following is obtained.

$$Zs = \frac{Z_1}{2} + \frac{Z_1Z_2+2Z_2Zs}{2Z_2+Z_1+2Zs} =$$

$$\frac{2Z_1Z_2+Z_1^2+2Z_1Zs+2Z_1Z_2+4Z_2Zs}{4Z_2+2Z_1+4Zs}$$

$$4Z_2Zs+2Z_1Zs+4Zs^2=$$

$$2Z_1Z_2+Z_1^2+2Z_1Zs+2Z_1Z_2+4Z_2Zs$$

$$4Zs^2=4Z_2Zs-4Z_2Zs+2Z_1Zs-2Z_1Zs+$$

$$4Z_1Z_2+Z_1^2$$

(cancellation)

$$Zs^2=Z_1Z_2+\frac{Z_1^2}{4}$$

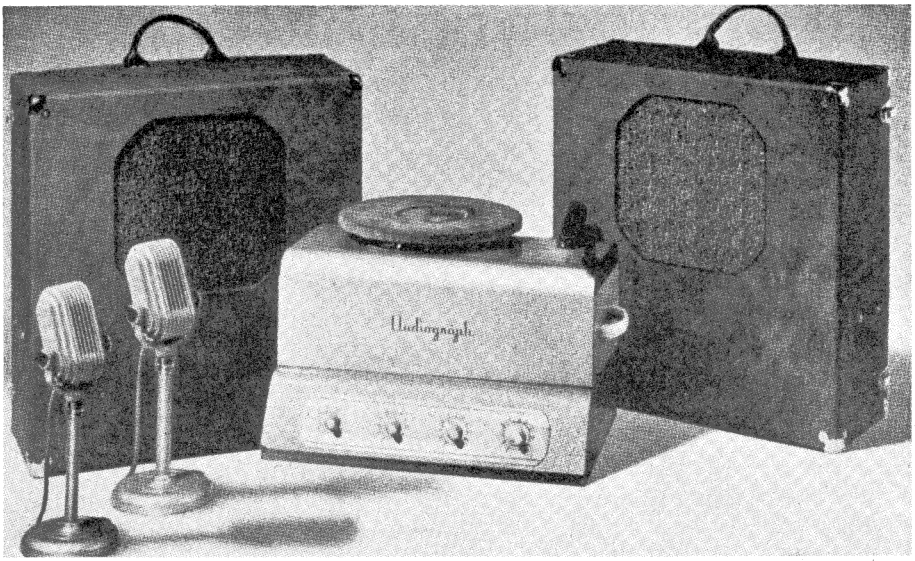
$$Zs=\sqrt{Z_1Z_2+\frac{Z_1^2}{4}} \qquad (4)$$

Thus it becomes possible to find the characteristic impedance of a line in terms of the series and shunt elements.

Since the terms *susceptance* and *admittance* must now be introduced, their meaning will be defined at this time. As you know, it is often more convenient to refer to the *conductance* of a circuit instead of its resistance. Thus, for vacuum tubes reference is made to the *plate conductance* instead of plate resistance in many cases. The term *mutual conductance* is more accurate as to merit and description of this characteristic than is mutual resistance of a tube. Resistance defines the amount of opposition the circuit will offer to a current or signal while conductance defines the signal or current carrying ability of the circuit. These terms are inversely related—that is, as the resistance increases, the conductance reduces and vice versa.

In the same sense, it is sometimes more convenient to consider a circuit as being *susceptible* to a certain AC signal or current being influenced by forms of reactance. This ability of a circuit to carry an AC signal is called the circuit *susceptance*. It is the reciprocal of the reactance in the circuit. For example, if the reactance is (XL+XC) the susceptance is $\frac{1}{XL+XC}$ or SL+SC or, if the reactance is simply defined as X, the susceptance would be equal to $\frac{1}{X}$ and this, in turn, may be symbolized as S just as $\frac{1}{R}$ is symbolized as G (conductance).

The ability of a circuit to carry a signal or AC current when influenced *by both resistance and reactance* (impedance) is called its *admittance*. If the impedance of a circuit is defined as Z, its admit-



This view shows typical equipment used in a medium sized Public Address system. In the background are shown two portable speakers with two microphones shown at the left. The center unit houses the AF amplifier and its accompanying turn table and pick-up arm.

tance is $\frac{1}{Z}$ and may be symbolized as *Y*. It will be convenient to remember that susceptance (S) is the reciprocal of reactance or $\frac{1}{X}$ and that admittance (Y) is the reciprocal of the impedance or $\frac{1}{Z}$.

Now for a very short length of transmission line, the *series impedance* being proportional to the length will be equal to the *product of the length and the series impedance*. The short length will be identified as dL, while the series impedance will be identified as $Z_1 \times$ dL. On the other hand, the *shunt impedance* of the line will be the reciprocal of the shunt admittance for a short length also proportional to the length and may be written $\frac{1}{YdL}$. Because of its relation to formula No. 4, it must be regarded as an admittance rather than an impedance. Substituting these values in formula No. 4 the following is obtained:

$$Zs = \sqrt{\frac{Z_1 dL}{YdL} + \frac{Z_1 dL^2}{4}}$$

This equation cannot be evaluated in the foregoing form without knowing the value of dL. The other values may be calculated readily as will be explained. Further, an extremely short length of line is being considered here or one where the value dL approaches zero.

The value of Zs in this equation (when dL is made exceedingly small) approaches the value $\frac{Z_1}{Y}$ as dL approaches zero. The terms dL in the Zs equation cancel in the first term under the radical (square root sign and the second

term $\frac{Z_1 dL^2}{4}$ approaches zero.

These quantities may now be expanded so that they may be evaluated. You have seen that the *series impedance* of a transmission line is made up of resistance and inductive reactance in series which always have the relation—

$$Z_1 = \sqrt{R^2 + (2\pi fL)^2}$$

Where: R = series resistance.
L = series inductance.
f = frequency.

The admittance, of course, is made up of the shunt leakage conductance and susceptance due to capacity. It will be recalled that susceptance is the reciprocal of reactance and that the reactance, in this case, is capacitive. These factors are related to Y as follows:

$$Y = \sqrt{G^2 + (2\pi fC)^2}$$

Where:

G = shunt conductance or $\frac{1}{R}$

C = shunt capacity
$(2\pi fC)$ = shunt susceptance
Y = shunt admittance

Substituting these values in the equation just developed, the following results:

$$Zs = \sqrt{\frac{\sqrt{R^2 + (2\pi fL)^2}}{\sqrt{G^2 + (2\pi fC)^2}}} = \sqrt{\frac{Z}{Y}} \qquad (5)$$

This algebraic equation is in the so-called rectangular form and capable of quantitative evaluation directly by substitution of values. The qualitative form sometimes called the *polar* form reads as follows:

$$Zs = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

(See your SAR Math Book for more information.)

This latter form is presented here so that you will recognize the similarity and relation of the two in technical work. Equation (5) is often simplified into the following form:

$$Zs = \sqrt[4]{\frac{R^2 + (2\pi fL)^2}{G^2 + (2\pi fC)^2}}$$

Also the following form may be used:

$$Zs = \sqrt[4]{\frac{R^2 + \omega^2 L^2}{R^2 + \omega^2 C^2}}$$

This formula (5) may be put to practical use by determining Zs from known constants. An open wire line having its wires .165 inch in diameter has a DC resistance of 4.14 ohms per mile. Its inductance calculated from accurate bridge measurements is 3.64 millihenries per mile and its capacity between leads for 1 mile is .00845 mfd. Its average conductance per mile is .8 micromhos although this varies considerably with weather and temperature. Such a line consists of two conductors spaced 12 inches apart and is a standard telephone line. A frequency of 1000 cycles is assumed for the following examples.

Z equals:

$$\sqrt{(4.14)^2 + (6.28 \times 1000 \times .00364)^2}$$
$$= \sqrt{17.15 + 523}$$
$$= \sqrt{540.15}$$
$$Z = 23.25 \text{ ohms}$$

Note $Z = \sqrt{R^2 + (2\pi fL)^2}$

By use of trigonometry, it may be shown that an impedance of 23.25 ohms in this case will produce a *phase lead* of 79 degrees and 45 minutes. This will be referred to later on.

The value of Y or for the admittance is obtained as follows:

Y equals:

$\sqrt{(.8\times10^{-6})^2+(6.28\times10^3\times845\times10^{-5})^2}$

To solve this problem, first solve for the value in the right hand parentheses as follows:

$$(6.28\times10^3\times845\times10^{-6})^2$$
$$=39.47\times714{,}025\times10^{-4}$$
$$=39.47\times71.4$$
$$=2818.158$$

The left hand parentheses value is equal to:

$$(.8\times10^{-6})^2$$
$$=.64\times10^{-12}$$

The entire problem is now equal to:

$$\sqrt{.64\times10^{-12}+2818.15}$$
$$=\sqrt{2818.798\times10^{-12}}$$
$$=53.07\times10^{-6}$$
$$=.00005307 \text{ (admittance)}$$

(Note: $Y=\sqrt{G^2+(2\pi fC)^2}$

Now by use of trigonometry, an admittance of .00005309 is equal to a *phase lag* of —89 degrees and 5 minutes.

As stated, the source impedance or Zs is equal to:

$$Zs=\sqrt{\frac{Z}{Y}}$$
$$=\sqrt{\frac{23.25}{.00005307}}$$
$$=\sqrt{438{,}000}$$
$$=662 \text{ ohms}$$

The same form of calculation by trigonometry shows that for a Zs value of 662 ohms, there will be a phase lag of −4 degrees and 40 minutes.

Thus it is found that the characteristic impedance of the transmission line is 662 ohms. Moreover, you have seen by following through with the phase of the signal voltage as it has traversed the mile of line that the voltage lags the current, making the line have a slightly capacitive characteristic.

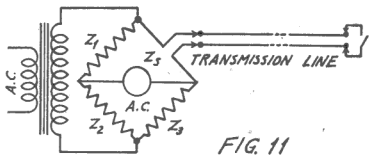


FIG. 11

The angle of lag is −4°40′ (minus 4 degrees and 40 minutes).

Later this will be referred to again to find the velocity of the signal over the line and its attenuation from these figures. Consider now another way of finding the impedance of a transmission line.

With a given length of line, assume the two ends of the line are attached to a bridge circuit as in Fig. 11 and assume the bridge is balanced with the remote end open.

Compute the impedance value from:

$$\frac{Z_1}{Z_2}=\frac{Zs}{Z_3} \qquad (6)$$
$$Zs=\frac{Z_1Z_3}{Z_2}$$

Now assume the switch is closed at the remote end and assume the bridge is balanced again—obtaining a *new value for Zs.* Call the first value Zs_1 and the second Zs_2 so that they may be distinguished. In equations 1 and 2, there are expressions for open and closed lines. Having found the characteristic impedance, the section may be loaded with this value, and you may solve for the input impedance which is also the characteristic impedance. Expressing this in terms of the line constants, as was done in equation (4) and by dividing this equation by the closed circuit impedance, the open circuit impedance will be obtained and by dividing it by the open circuit impedance the closed circuit impedance will

be obtained. This simply means that the square root of the product of the open and closed circuit impedances is equal to the *characteristic impedance* of the line. To prove this you can multiply equations (1) and (2) as follows:

$$\left(\frac{Z_1}{2}+\frac{Z_1Z_2}{Z_1+2Z_2}\right)\times\left(\frac{Z_1}{2}+Z_2\right)$$

$$=\frac{Z_1^2+4Z_1Z_2}{2Z_1+4Z_2}\times\frac{Z_1+2Z_2}{2}$$

$$=\frac{(Z_1+2Z_2)\ (Z_1^2+4Z_1Z_2)}{4(Z_1+2Z_2)}$$

$$=\frac{Z_1^2}{4}+Z_1Z_2$$

Therefore:

$$Zs^2=\frac{Z_1^2}{4}+Z_1Z_2=Zs_1\times Zs_2$$

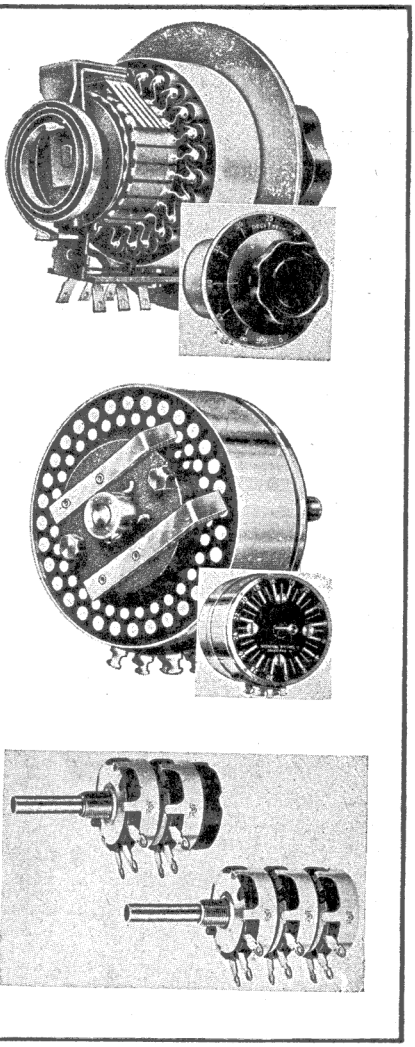
and— $$Zs=\sqrt{Zs_1\times Zs_2} \qquad (7)$$

You may easily test the correctness of this using the former values which were:

Open circuit impedance
—1100 ohms
Short circuit impedance
—199 ohms

and— $Zs=\sqrt{1100\times199}=\sqrt{218,900}$
$=468$ ohms

Any signal source feeds a transmission line a given amount of energy. The line, in turn, gives this energy to the load. For example, if the line were sufficiently long it could be supplied with a signal and if the source was disconnected the load would still receive all the signal fed to the line. Moreover, from a small fraction to several cycles may be in the line at various distances from the source, depending on the length of the line. If the line is two wavelengths long and it was supplied with exactly 2 cyles of the signal and if the signal source was removed before the first part of the signal arrived at the load, the complete signal would traverse the line. This, of course, would be true with any length of line, but if it is very short, there would be only a small fraction of difference between the phase of the signal sent and the one arriving at the load. When this goes so far that



Here are shown various types of pads or attenuators as mentioned in the latter part of this lesson. The two units at the top and center employ fixed resistors—the variable value being obtained by means of switches. These are similar to the schematic form shown in Figs. 18 and 19. The lower types are continuously variable and are represented by Figs. 24 and 25.

the phase between the two voltages is less than .25 degree difference or less than .5% as measured in amplitude, the characteristic impedance becomes unimportant.

As an example, if you have a piezo-electric pick-up with an impedance of 150,000 ohms and wish to connect it to the grid input of a tube with a line 6 or 8 feet in length, you need not consider the impedance of the line—that is, the line need not be matched with transformers. The line may have only 200 ohms impedance (usually true in practice) and yet the results are, for all practical purposes, perfect. No transformer or other matching apparatus is needed in this case.

If the line is shielded, the velocity of wave motion over it is somewhat lower, and it may be a factor at very high frequencies. For this reason it is well to consider this matter of the phase shift between the source and load for various lines. Until the subject is explained more thoroughly, assume the velocity of the wave for an open wire line to be about 180,000 miles per second. Assume that it is desired to transmit 14,000 cycles from the pick-up unit. The wavelength for these conditions would be a little over 12.85 miles. The phase changes 90 degrees or from maximum in .25 cycle which would be .25 of 12.85 or 3.21 miles. With this length or longer, the source line and load impedance must match for best results, but as the line become shorter than this, a successive deviation from the line impedance value may be tolerated. A phase change of .25 degree would take place in a distance of 1/360 of 3.21 miles which is .00891 miles or 47.1 feet. If the line were shielded, the propagation rate would be reduced considerably and the tolerable length for satisfactory transmission would be reduced by the same factor. The other reasons for making a line shorter than 50 feet, for example, is because of DC loss in extremely low impedance circuits, low signal levels where the attenuation would make the signal compare with the noise level or extraneous pick-up through induction into high impedance circuits.

Of course, in high frequency work where the signal phase changes 90 degrees in a few feet of line, you must match impedances very accurately to the line as well as to the load. You can easily see that the difference in all reactances in this case would be enormous.

For a dynamic or velocity microphone, a transformer may be used directly at the microphone not because the line is going to have a serious impedance effect on the signal but because the signal is of the high current type, and its attenuation, due to DC resistance, otherwise would be 1 db every few feet. In the case of the crystal microphone, the original level is −120 db and any further attenuation would bring it down to the thermal agitation level (noise level) for copper wire. This is the level at which atomic vibration loosens enough electrons in the wire to form voltages comparable with the signal at room temperatures. As you know, the carbon microphone can be led several hundred feet through almost any cable

as its impedance is within 50 or 100% of that of the line anyway. The limitation here is principally one of DC voltage drop. With the crystal microphone an open line approximately 50 feet long may be used and the signal may be fed directly into the grid of a vacuum tube with an impedance of 150,000 ohms although the *characteristic impedance* of the line may be only 80 ohms. In spite of the line, the source and load must match as closely as possible. This length of line is not ordinarily used because of the possibility of pick-up by induction or of the necessity of shielding, etc., and not because of the mismatch of impedance. As long as a transformer must be used for other purposes, it may as well match the line to avoid other troubles mentioned.

From the known constants of a transmission line, it is possible to calculate the attenuation of any signal and its velocity. If you will again return to Fig. 9, it will be noticed that a certain signal current enters the transmission line at Zs, flowing toward ZL where it divides between the shunt and series sections. With the load ZL equal to the characteristic impedance of the line a lesser current I_2 will flow from here into the next section. It will again divide, shunting the same percentage of current as before across the line. The currents will be related in such a way that

$$\frac{I_1}{I_2} = \frac{I_2}{I_3} = \frac{I_3}{I_4}, \text{ etc.}$$

This constant ratio is the *reciprocal* of the loss or attenuation of one section of the line. Since all other sections will load the first

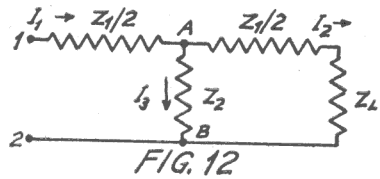


FIG. 12

one with an XL value, you can find the value of $\frac{I_1}{I_2}$ for one section, and this will be the same for each section. Then, by computing the constants of a section representing any unit length of line, such as a yard or a mile, you may find the attenuation for the entire line simply by multiplying by this factor. In a single section, as in Fig. 12, the ratio of input current I_1 to that entering the second section represented by ZL is the reciprocal of the ratio of the total from A to B, to only that taking the path through Z_2.

Thus—

$$\frac{I_1}{I_2} = \frac{\frac{ZL}{2} + Zs + Z_2}{Z_2}$$

As the total impedance from A to B is—

$$\frac{Z_2\left(\frac{Z_1}{2} + Zs\right)}{Z_2 + \frac{Z_1}{2} + Zs}$$

and that through which I_2 flows is—

$$\frac{Z_1}{2} + Zs$$

Thus—

$$\frac{\frac{Z_1}{2} + ZL}{\frac{Z_2\left(\frac{Z_1}{2} + ZL\right)}{Z_2 + \frac{Z_1}{2} + ZL}}$$

$$= \frac{Z_2 + \frac{Z_1}{2} + Zs}{Z_2} = \frac{I_1}{I_2}$$

Simplified, this equals—

$$1 + \frac{Z_1}{2Z_2} + \frac{Zs}{Z_2} = \frac{I_1}{I_2}$$

Eliminating the characteristic impedance from the equation by expressing the last term involving Zs in terms of the network impedance, you may obtain from equation (4)—

$$Zs = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}}$$

and dividing by Z_2—

$$Zs = \sqrt{\frac{Z_1 Z_2}{Z_2^2} + \left(\frac{Z_1}{2Z_2}\right)^2}$$

$$Zs = \sqrt{\frac{Z_1}{Z_2} + \left(\frac{Z_1}{2Z_2}\right)^2}$$

making the entire value of—

$\frac{I_1}{I_2}$ equal:

$$1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \left(\frac{Z_1}{2Z_2}\right)^2} \qquad (8)$$

This is the complete ratio of I_1 to I_2 in terms of the constants of any of the sections. You can substitute values in this equation and find the attenuation of any line. Being expressed in impedance, it should be obvious that in addition to the currents actually reducing they also change phase and that this equation involves both factors. That is, these factors are both working to produce the total attenuation experienced by the signal. The equations for the current ratios for various sections of the line show the attenuation to be logarithmic in nature; hence, the entire right term of (8) must be the result of raising some number to some power. If you start by choosing any number as the base, an appropriate power must be affixed to it for this value while, if you choose any power, a number can be found which will satisfy the equation. To keep the entire equation in mathematical unity, it is most convenient to use the term epsilon *e* (see your SAR Math Book for an explanation of *e*) as the base and find its exponent which will satisfy the equation. Thus—

X equals:

$$\log e\left(1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \left(\frac{Z_1}{2Z_2}\right)^2}\right)$$

as—

$$e^x = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2} + \left(\frac{Z_1}{2Z_2}\right)^2}$$

While it is beyond the scope of this study to go into the *binomial theorem* for the expansion of this third term for evaluating x in the above equation, the result of this operation will show that x will have a value of—

$$\sqrt{ZY}$$

or the exponent for *e* to give the above value in terms of the general line characteristics would be written—

$$e\sqrt{ZY}$$

From you SAR Math Book you will find that *e* is equal to 2.71828. In an example in the foregoing part of this lesson for a typical line it was shown that Z was equal to 23.25 with a leading phase of 79 degrees 45 minutes and that Y was equal to .00005307 with a lagging phase of minus 89 degrees and 5 minutes. Using this same transmission line in the next example the attenuation and phase change of the signal per mile of line can now be obtained. In another ex-

ample the phase value was obtained but no use was made of it. It may now be applied as explained in the following. First, the impedance is multiplied by the admittance. Then the square root of this product is extracted and the resultant phase is found by dividing the sum of the two angles by 2. An explanation follows:

$$e\sqrt{ZY} = e\sqrt{23.25 \times .00005307}$$
$$= e\sqrt{.0012338775}$$
$$= e \times .03512$$
$$= 2.7182 \times .03512$$
$$= .095463184 \text{ or } .09546$$

Thus the attenuation constant for the line is equal to .09546.

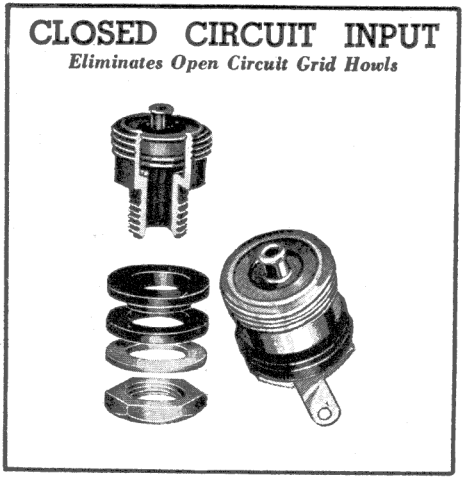
As pointed out, trigonometry shows that a source impedance of 23.25 ohms has an angle of 79 degrees and 45 minutes and that an admittance of .00005307 micromhos has an angle of 89 degrees and 5 minutes. The sum of these is equal to 168 degrees and 50 minutes and division by 2 gives a value of 84 degrees and 25 minutes. Further use of trigonometry (which, because of its complexity, will not be repeated here) shows that the phase change factor for the line is .033526.

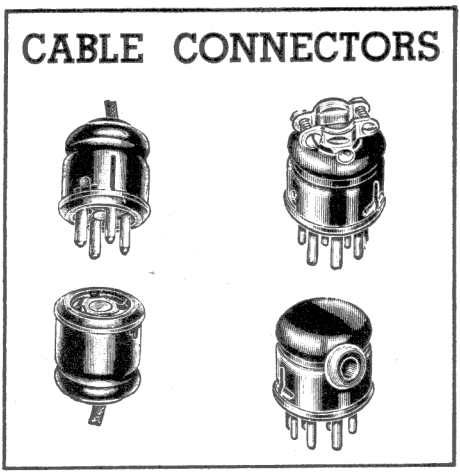
These figures were all obtained from basic mathematics, and they may not mean much at the moment, but the following explanation will show how they are used. A standard telephone line has been found to reduce the voltage as applied to its input approximately 36.8% per mile. This is a reduction of (e) 2.7182 times per mile approximately, and, since the relation is logarithmic, this value was used as the unit of measure of attenuation just prior to the use of the more convenient unit, the decibel, now in wide use. From these figures it will be seen that a voltage ratio of 2.7182 is equivalent to 8.686 db as it is 20 log 2.7182. The dimension was not a mile but a distance which would produce exactly this attenuation. Later the decibel was derived electrically and not from practice.

When the attenuation was 2.7182 or equal to 36.8% of the original, the attenuation factor was called one *neper*. Thus, one db is equivalent to 8.686 nepers. It may be noticed that the figure 2.7182 is (e) the base of *naperian logarithms* used for simplifying the mathematics of the problem. After the proper factors were found, as in the foregoing, other values may be found easily by conversion. For example, the number .010482 is the attenuation factor in nepers per mile. In this case, it is a real mile. This is equal to .09546 db per mile. Considering that it would take 63 miles to cut the signal down 6 db or in half (50%) approximately, it is a remarkably efficient line.

Now consider the value obtained as the phase change factor. This gives the fraction of an angle per mile. The unit of measure of this angle is called the *radian* and is equal to approximately 57 degrees 20 minutes. It is exactly equal to $360/2\pi$. In other words, there are exactly 2π radians in a circle of 360 degrees. This means that the phase change is .033526 radians per mile, which, divided into 2π, gives 187 miles per cycle and at 1000 cycles there would be 187,000 miles per second for the velocity of the wave on the transmission line.

It is often convenient to get the





The above view shows two types of microphone or phonograph pick-up accessories. The unit in the box to the left shows a special type of input to the grid of an AF amplifier. When no input is being used, this unit keeps the grid circuit of the tube closed. In this way, normal bias is applied to the grid. When a screw type connector is attached, the input jack opens, allowing the connection of a mike or pick-up The units in the box to the right show different types of cable connectors for speakers, microphones, etc.

characteristic impedance of a line from its dimensions instead of its electrical characteristics. This is done by simply converting all of the values in an expression for the impedance in electrical terms to those in dimensional terms and solving for the impedance. This work is quite elaborate and there is no exact solution. A very satisfactory approximation can be obtained—probably as close as the line can be build in ordinary cases from the following formula:

$$Zo = 277 \log \frac{2s}{d}$$

(log base 10 used here)

Where—

s=the spacing of the wires.

Zo=characteristis impedance of the line.

d=the diameter of one wire.

(Same units of measure for both)

To demonstrate the simplicity of the use of this formula, suppose you have an ordinary twisted pair line such as would be used for a carbon microphone. Assuming that the diameter of the wire averages for the strands .25 mm. and that the spacing is 4 mm., there is obtained, by substitution, the following:

$$Zo = 277 \log \frac{2 \times 4}{.25}$$

$$= 227 \log \frac{8}{.25}$$

$$= 277 \log 32$$

$$= 277 \times 1.50515$$

$$= 417 \text{ ohms approximately.}$$

This equation for a concentric line or a coaxial line is—

$$Zo = 138.5 \log \frac{ro}{ri}$$

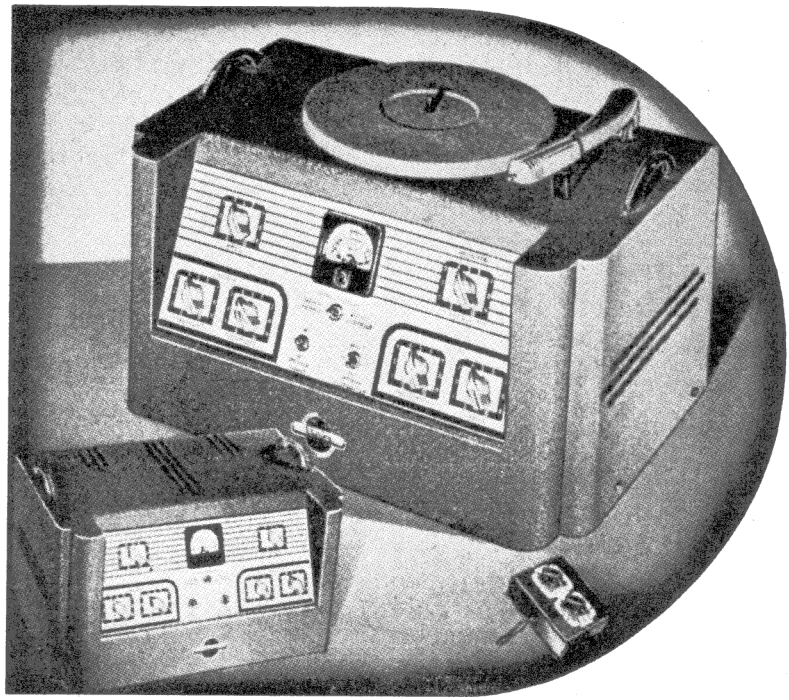
Where—

ro=inner radius of outer conductor.

ri=outer radius of inner conductor.

WAVE REFLECTION

The study of wave reflection will give additional information on how a transmission line can introduce reflection distortion when



This photo shows a very common type of AF amplifier. Such a unit may be obtained with and without the turn table and pick-up unit. Such a system as this would employ input transmission lines for microphones and output transmission lines for speakers.

matching transformers are not used. In any reactive circuit there will always be a certain amount of energy stored in the electrical units as, for instance, the electric field (voltage) or the magnetic field (magnetism). You have seen in previous studies that once these units are provided with energy they can give it back to the circuit when the energy source is removed. Thus, if you feed a certain amount of energy to a transmission line which always contains reactance, a certain amount of it will be *stored* in the line. Another portion of it will be stored in the load.

If the line is a quarter of a wavelength long based on the velocity of the wave, all of the input energy will, at first, be stored in the line and, at a later time, all will be stored in the load. If the line is ⅛ wavelength long, not more than 70.7% of the total input energy can be stored in the line at any time. This is because the greatest difference in amplitude in an ⅛ cycle length of a sine wave can never exceed 70.7% maximum. If the line is 1 degree long (1/360 wavelength), only 1.75% of the total energy can be stored in the line. The balance of the transmitted and used energy can be stored in the load—in this case, it is 98.25 per cent.

Now in long lines (¼ cycle or longer), the energy reflected from the load into the line is most important while in a short line, the energy reflected from the load to

the source is most important. In either case, the load will be fed from an impedance of the line or an equal one from the source, provided the source impedance matches the line. If the load impedance matches the line or source impedance (taking into consideration both cases) the signal will continue into the load without interruption as though the line were indefinitely long, but if the load impedance is either larger or smaller than the line or source impedance, *a portion of the energy will be reflected back to the source.* It will be attenuated and its phase will be shifted just as for the initial signal as it travels toward the source. Since the load does not match the source impedance, the load will create a signal from its stored energy which, in turn, does not match its source, and it will then be again reflected combining with the original signal. The signal is decreasing toward the source, and by the time it again reaches the source, after being reflected, it may, be ,quite low in value. The longer the line, the lower it will be. However, the reflected signal current is largest at the point of reflection where it may have the maximum effect on the approaching signal.

A portion of the energy of the total reflected bears a definite relation to the difference between the load impedance and the impedance feeding it. This relation may be stated in the form of a ratio as follows:

$$\frac{Ir}{Ii} = \frac{Zo-ZL}{Zo+ZL}$$

Where—

Ir=reflected signal current.

Ii=incidental or first transmitted signal current.

To illustrate this, assume a value of 1000 ohms for Zo and let ZL vary from 100 ohms to 1000 ohms evaluating the initial ratio at various points. The following table shows the values assumed and those found for each value assumed.

ZL	Ir/Ii
100	.818
200	.666
300	.538
400	.429
500	.333
600	.250
700	.1762
800	.1111
900	.0526
950	.02565
975	.01266
990	.005025
1,000	.0

Here it is obvious that when the impedances are equal there is no reflected signal while with 975 ohms connected to a 1000 ohm source or line, there is a reflected current of 1.266% of the signal current delivered to the load. This is the ratio of the two currents at the load.

PADS AND VOLUME CONTROLS

You are familiar with the purpose of a volume control. However, volume controls for transmission networks and lines differ radically from those used in receivers. They must comply with many more requirements than those for receivers. A pad is a *non-reactive* element inserted in a transmission line or network for the purpose of introducing a definite loss in the

line. It is very often convenient to reduce the signal at certain points in a network in this way. Tubes and amplifiers have definite gain values when operating at best efficiency and to reduce gain by omitting efficiency often produces distortion which the pads do not create. For other reasons, it is not only convenient but essential to reduce volume by a fixed amount by this means.

Having developed the equivalent *T* theorem for transmission lines in the first part of this lesson, you should be familiar with the *T* circuit as a section of a transmission line. Any transmission line may be matched at one frequency with a *T* section inserted anywhere along its length. Furthermore, the *T* section can be made to dissipate any portion of the energy of the signal without changing its input or output impedance. For any value of line impedance the ratio of current leaving the *T* section to that entering the section is, of course, determined by the series and shunt impedance values of the section. Naturally, if the series elements are small and the shunt elements large, there will be little difference in the two and their ratio will be small. On the other hand, if the series elements are large and the shunt elements are small, the loss will be relatively great. The exact relation of the ratio which may be called *R* and the characteristic impedance Zo in terms of the series and shunt elements will be equal to:

$$\text{Series Element } Z_1 = Zo\frac{R-1}{R+1}$$

$$\text{Shunt Element } Z_2 = Zo\frac{2R}{(R-1)(R+1)}$$

In this work, absolute values of current or voltage for loss or gain is not considered but rather the ratio of one voltage or current to the other in *db* rather than in a fraction. In this way the amount of loss can be calculated in the greatest possible steps which can be tolerated in the final reproduction without notice. For example, the ear can just perceive an increase or decrease of signal voltage or current of approximately 12%, which is almost exactly 1 db. One-hundred per cent increase or 50% decrease—that is, to double or one-half, is almost exactly 6 db.

Now suppose that you wanted to place a pad in a 500 ohm line as in Fig. 13, having values such that each end would exactly match the line and yet the signal would be cut exactly in half or 6 db. First it is necessary to convert the ratio term *R* into an expression of db so that substitutions can be made directly in the formula without the preliminary work of finding the db equivalent of each ratio first.

Thus—

$$db = 20 \log R.$$

$$\log R = \frac{db}{20}$$

$$R = \text{antilog}\frac{db}{20}$$

—also written $\log^{-1}\frac{db}{20}$

(Log to the base 10 is used throughout in this work)

This value can then be substituted

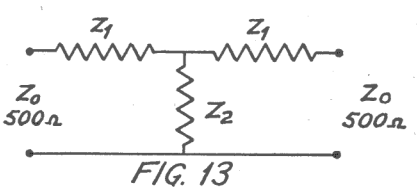


FIG. 13

in place of R in the equation as follows:

$$Z_1 = Z_o \frac{(\log^{-1}\frac{db}{20}) - 1}{(\log^{-1}\frac{db}{20}) + 1}$$

$$Z_2 = Z_o \frac{2(\log^{-1}\frac{db}{20})}{(\log^{-1}\frac{db}{20} - 1)(\log^{-1}\frac{db}{20} + 1)}$$

These equations are the final preparation for finding the values of the shunt and series elements of any *T* section for any attenuation for matched sides (that is, where the input matches the output).

In the example a 500 ohm line has been chosen in which a pad is placed which will cut the signal in half or by 6 db. Substituting this value in the first equation the following is obtained:

$$Z_1 = 500 \frac{(\log^{-1}\frac{6}{20}) - 1}{(\log^{-1}\frac{6}{20}) + 1}$$

$$Z_1 = 500 \frac{(\log^{-1} .3) - 1}{(\log^{-1} .3) + 1}$$

Now the number .3 is the log of a number which it is desired to find. If you examine a log table, you will see that opposite this number in the table is the number 1.995. In other words, log 1.995 equals .3. Therefore, the antilog (number) of .3 (log) is 1.995 which must now be placed in the equation for further evaluation.

$$Z_1 = 500 \frac{1.995 - 1}{1.995 + 1}$$

$$= 500 \frac{.995}{2.995}$$

$$= 500 \times .333$$

$Z_1 = 166.6$ ohms.

With these values substituted in the equation for the shunt element, the following is obtained:

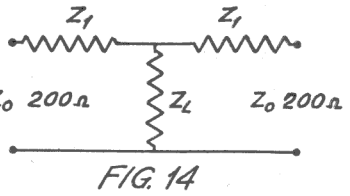


FIG. 14

$$Z_2 = 500 \frac{2 \times 1.995}{(1.995 - 1)(1.995 + 1)}$$

$$= 500 \frac{3.99}{.995 \times 2.995}$$

$$= 500 \frac{3.99}{2.98}$$

$$= 500 \times 1.34$$

$= 670$ ohms

Quite frequently these values are tedious to work out for standard lines and pads, and so tables are usually available such as the one following, giving the proper values for series elements and shunt elements to be used with them for a standard line. The table applies to a 200 ohm line and includes every value from 1 db through 15 db. See Fig. 14.

db	Z_1	Z_2
1	11.5	1737
2	11.85	1657
3	13.2	1513
4	14.67	1355
5	16.35	1215
6	18.1	1098
7	20.15	991
8	22.3	885
9	24.7	796
10	27.3	719.5
11	30.15	650
12	33.2	586.5
13	36.24	530
14	40.75	480
15	43.9	393

With the foregoing formulas other tables may be made for similar problems.

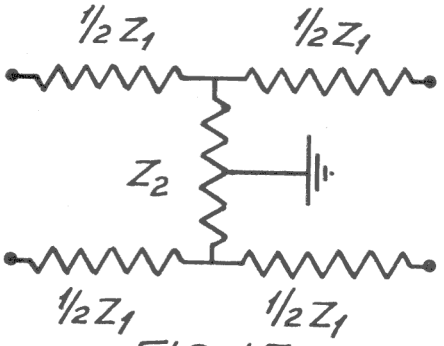


FIG. 15

In some cases it is necessary to keep both lines of the complete circuit away from ground potential. Such a line is called a balanced line and two more arms are placed in the other lead, as in Fig. 15. For the values for this type of pad, simply put half of each of the series arms in the other side of the line. Their values will, therefore, be $Z1/2$. The values of the shunt arm do not change.

In some cases ,the electrical network demands a Pi (π) section either for electrical convenience or because the reduction by mathematics is simpler. In this case for the two shunt elements, twice the value of each are used for each db value and the two series arms are simply considered as one unit having twice the value of Z_1. This is indicated in Fig. 16.

It is often desirable to have both sides of a transmission line free from capacity to ground. That is, each side of the line will have the same capacity to ground. This is particularly necessary where *phantom lines* are use. A phantom line is one which uses the two conductors of one line for one conductor and the ground for the other, as in Fig. 17. In this case, it is evident why the line must be balanced.

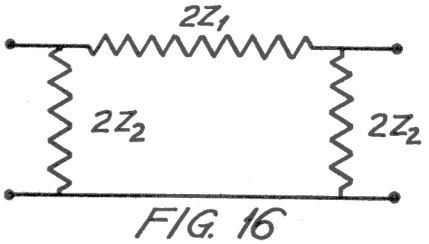


FIG. 16

To properly pad a line of this kind, an *H* pad is used as in Fig. 15. At certain places it is grounded in the center while at others a ground is not necessary. All values for this type of pad are figured just as described for a *T* pad and each value of series element is used with half of it in one side and half in the other, as in Fig. 15.

While cutting down the signal value a definite amount, it is often

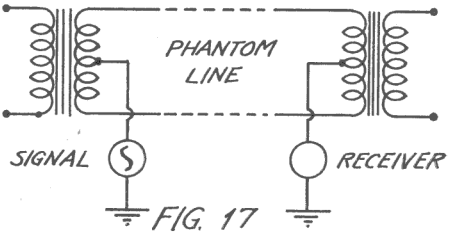


FIG. 17

desirable to match two lines of different values to its ends. For example, it may be desirable to attach a 200 ohm line to a 500 ohm line and, at the same time, reduce the energy by 8 db. Formulas for an *H* pad for this work are given as follows:

$$Z_1 = Z_3 = Zx - \left(Z_5 - \log^{-1}\frac{db}{20}\right)$$

$$Z_2 = Z_4 = Zy - \left[Z_5 - \left(\log^{-1}\frac{db}{20} - \frac{Zy}{Zx}\right)\right]$$

$$Z_5 = \frac{2\log^{-1}\frac{db}{20}\sqrt{Z_1 Z_3}}{\left(\log^{-1}\frac{db}{20}\right)^2 - 1}$$

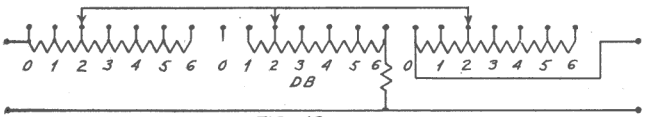


FIG. 18

There are, of course, numerous other types of pads which are variations of these basic ideas. A lengthy text book could well be devoted to these variations.

VOLUME CONTROL

You have seen how volume can be reduced by a fixed amount simply by choosing the proper values for pads of various kinds. Extending this idea into variable units, a volume control as in Fig. 18 for a *T* pad may be converted into a volume control simply by making its elements variable so that various values may be selected. Each value is chosen for any type so that the volume will be decreased or increased 12.2% or 1 db. This is the percentage of the signal at any level that it happens to be. For example, if the signal voltage is 100 volts at the point of the volume control, it may be cut down to 89.1 volts or up to 112.2 volts from a medium point. Or, if the signal is at a level of 100 millivolts it may be cut to 89.1 millivolts or raised to 112.2 millivolts. Being in db it is a proportional cut rather than a linear cut in quantity as you have learned.

In an ordinary *T* volume control with three moving contacts as shown in Fig. 18, each value may be taken from the table in the foregoing to make the entire unit. If two moving contacts are more desirable than three, which is true in most cases, the same control can be arranged as in Fig. 19.

Sometimes it is desirable to substitute a single variable section for the two series elements of a plain *T* section volume control so that only one variable will be needed rather than two. The *bridged T* control, as it is called, is shown in Fig. 20. The symbol for a *T* volume control is shown in two ways—these are in Figs. 21 and 22. A control for varying the bridge element and the shunt element at the

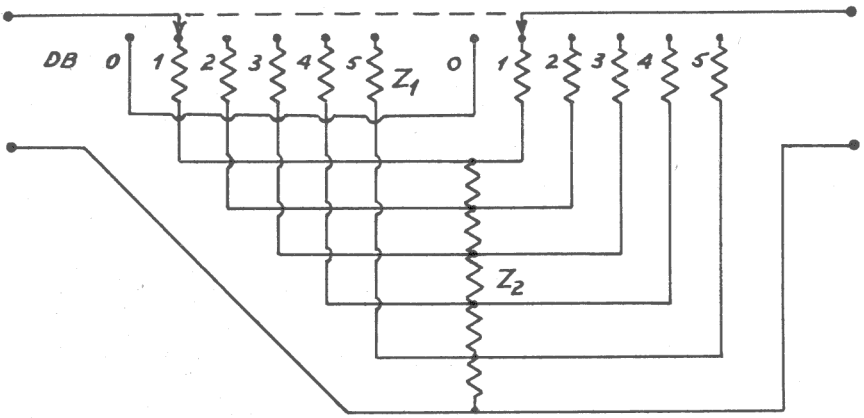


FIG. 19

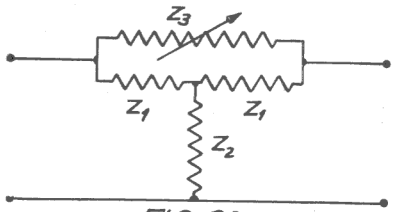


FIG. 20

same time is shown in Fig. 23 for the bridged *T* control.

Two interesting applications of the principles which have been covered in this lesson are shown in Figs. 24 and 25 respectively. In Fig. 24 at the top left, a 200 ohm signal source determined by the input impedance of the transformer feeds into a *T* volume control arranged for a 500 ohm input and an output which is equal to the entire network including R into which it

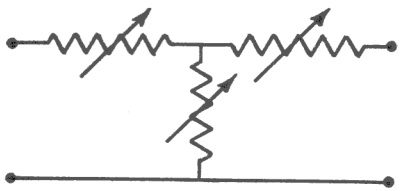


FIG. 21

faces. There is, therefore, no signal reflected into the source for any setting of the control. For the next source of 500 ohms, the *T* volume control drops the impedance of the source down to the same as for the one just discussed as it also must feed a signal into a circuit which will not reflect it.

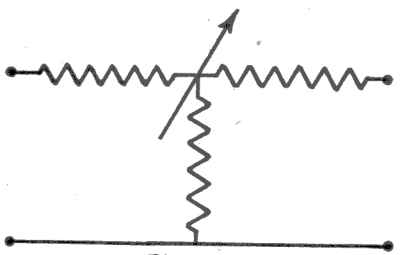


FIG. 22

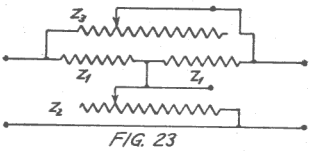


FIG. 23

The switches Sw are provided so that the circuit may be closed with the correct impedance if any of the signal sources are removed. Finally, an *H* volume control may be used as in Fig. 24 adjusted at its input to match the entire combination of source outputs and feeding into a matched 500 ohm master transformer. *This is known as a balanced series mixer.* A balanced shunt mixer which accomplishes the same thing is shown in Fig. 25.

LINE EQUALIZERS

All through this study certain aspects of frequency have been ignored which would affect calculations and put undue difficulty in them. The calculations would become so enormously difficult that a separate problem would have to be set up for each frequency in the audio spectrum just to study this insignificant part of the entire picture. Each frequency would have its own velocity of propagation, its own phase shift per mile, its own percentage of loss across the line, its own percentage of storage capacity in the line, its own reflection properties, etc. It is, therefore, necessary to choose generalities applying to certain groups of frequencies and treat them as approximations.

Disregarding the phenomena of resonance of a line with certain signals for the time being, it may be said that the effect of a line generally is to cause continuous attenuation or *progressive attenuation*

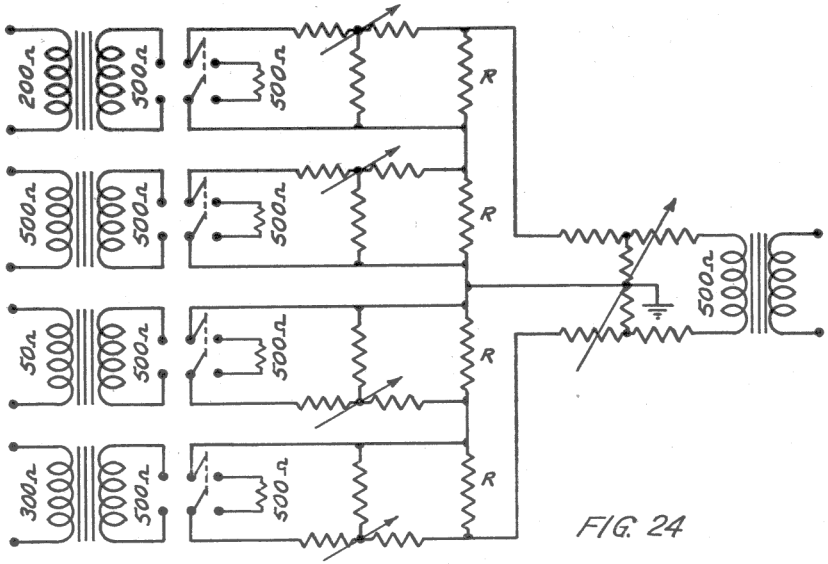


FIG. 24

for a certain band of frequencies. For example, the effect of a line on the audio spectrum would be somewhat as in Fig. 26. The attenuation increases (loss increases) as the frequency increases. This is principally due to the decreased reactance across the line for increased frequencies. However, this is a *progressive condition*—there being no peaks in the curve marked by resonance. Since a line has no way to amplify a signal, the only thing that can be done to it is to make all of the line loss as nearly equal to the maximum loss of the line for the highest frequency signal. Such a circuit must bring about a progressive loss in the signal as the frequency is lowered. The correct combination of inductance, capacity and resistance will accomplish this.

A circuit for accomplishing this is called an *equalizer*. In Fig. 27 is shown a simple but widely used

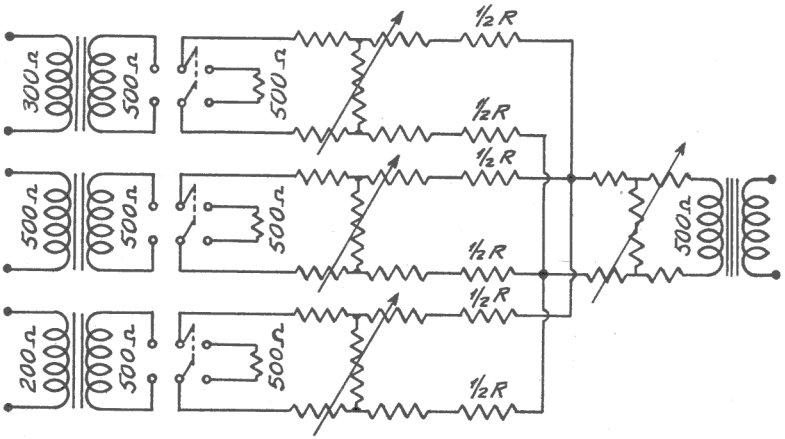


FIG. 25

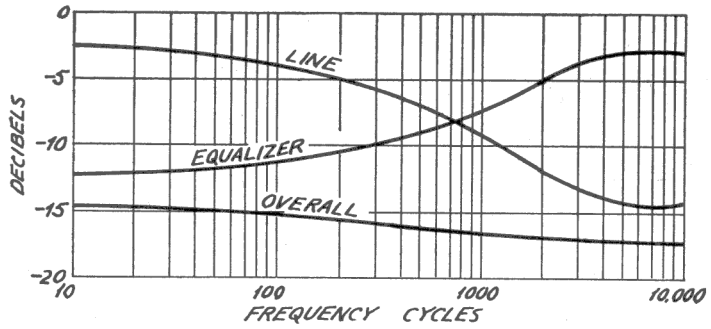


FIG. 26

form of the shunt equalizer. The resonant point of the L and C components of this unit should be above the highest frequency to be transmitted over the line and the resistance value is determined from the admittance of the equalizer itself and is the reciprocal of this value. In practice, the values are chosen by mathematics and then the values are adjusted for operation by a test of the line. The overall *admittance* of the equalizer reduces progressively with frequency producing a loss, as in Fig. 26. The value of R helps to determine the shape of the curve although it may never be the exact reciprocal of the loss of the transmission line, it may easily be a very small fraction of a db of it.

Where there is a pronounced tendency for the line to transmit one closely related group of frequencies better than all others due to resonance, a series equalizer is used as shown in Fig. 28. The resistance R is to broaden the curve of attenuation properly to fit the attenuation characteristics of the line as closely as possible. L and C are obviously made to resonate at the peak transmission frequency so that this frequency will be attenuated the most. Other combinations of resistance and L and C circuits in series and shunt with the line serve other less common or regular characteristics of the line.

It is often necessary to use filters at various points in the circuits to eliminate undesirable frequencies or to use the line for more than one purpose. There are two

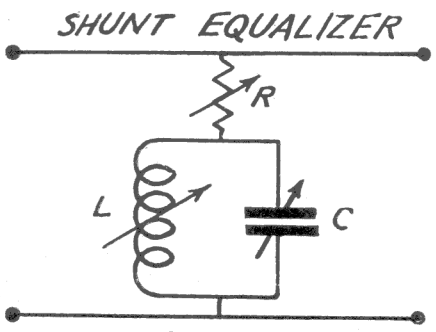


FIG. 27

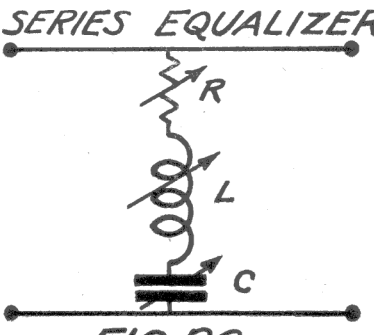


FIG. 28

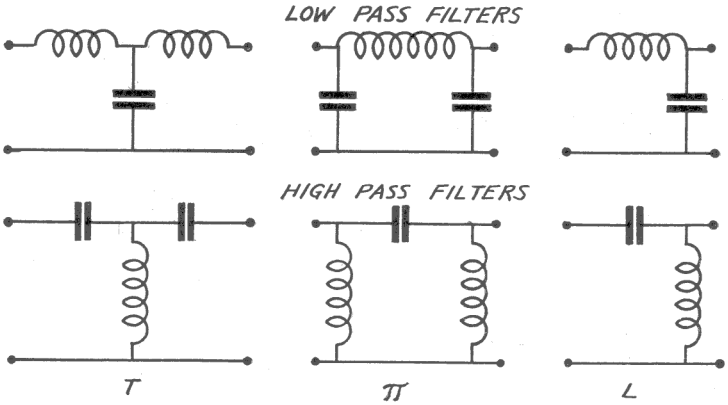


FIG. 29

general types of progressive filters, one to pass only progressively low frequencies, as in Fig. 29, and one to pass progressively high frequencies. Naturally, the series inductance is the low pass filter and the series capacity is the high pass filter because of the nature of reactance changing the ratio of series drop to shunt drop with frequency. There are also many types of resonance filters for passing or eliminating any desired band of frequencies. They are composed of combinations of resonant circuits as for the equalizer, both in series with the line and in shunt with it, as in Figs. 27 and 28.

PRACTICAL APPPLICATIONS

There is one basic reason for using a transmission line of a certain impedance regardless of the type of electrical energy to be transmitted. This basic reason goes back to some of your former studies on coupling. You have learned that, in order for the maximum amount of power to be transmitted from one unit to another, the coupling or connecting medium between the two circuits in question must have its impedance so adjusted that it matches both output and input units. Unless this condition can be brought about, there will be a power loss, and the system becomes inefficient. A practical example is a public address system, feeding a group of widely separated speakers. The power output transformer of this system might, for example, employ a secondary (which will feed the line) of 500 ohms impedance. It would then be mandatory to use a transmission line with a characteristic impedance of 500 ohms. Any pads, equalizers or attenuators connected to this line would also require a characteristic impedance of 500 ohms. Finally the transformers used to feed energy from the line to the voice coils would require a characteristic impedance of 500 ohms on their primary side. Any impedance values other than 500 ohms would unbalance the system, and it would not be efficient in transferring the energy, to say nothing of distortion and phase changes that might take place. This is a fundamental principal for you to remember no matter whether energy is being trans-

ferred from a battery, generator, vacuum tube, phonograph pick-up unit, microphone or other power source.

In addition to all the other uses for transmission lines as mentioned in this lesson, *carrier current* lines are sometimes employed wherein *carrier frequencies* are fed to wire lines, and these have superimposed on them many kinds of intelligence, such as speech, radio programs, telegraph signals, picture signals, etc. All such lines must be matched in impedance and oftimes repeater amplifiers must be placed along the lines to maintain the energy level. Where high fidelity must be maintained or where television or FM signals are involved the frequency considerations become highly important, and even a short line of just a few feet must be carefully watched and padded for efficient control and transfer of signal energy.

The normal voice range is from about 200 to 3000 cycles as for telephone conversation. A line from 1 to 2 miles long could handle this range of frequencies before the need for impedance matching would be encountered. For musical programs from about 100 to 6000 cycles, distortion would begin to be evident in about 3000 to 3500 feet, and for high fidelity with little or no frequency attenuation, distortion would begin to be evident in just a few feet of line.

For high frequencies, it is essential to use balanced lines of large solid conductors (with large surface area) or litz wire. You probably will have occasion to work with transmission lines in the high frequency field if you have contact with broadcast or amateur radio stations. Also in the receiver field, transmission lines are required for the all-wave receiver and in both television and FM receiving systems. It is necessary to feed the weak picked-up energy from the antenna to the receiver. A transmission line is usually used for this purpose. It will employ a matching transformer at the antenna and one at the receiver. In all cases, the recommendations of the manufacturer of the equipment should be followed no matter if AF or RF is involved. The manufacturer knows how to get maximum service from his product, and he or his agent for your community can advise you as to what type of line, coupling equipment, etc., is best for a given installation.

An attempt has been made in this lesson to cover a great deal of rather obscure material, bringing it into the practical scope of the serviceman. It is a highly specialized study and is rarely attempted by the practical engineer in other pursuits. The progressive student should know this material as well as possible, just as well as the glass blower in a tube plant should know something about the tube and its circuit to improve his work and become proficient. Many students have dismissed it as being out of their sphere of work only to turn to it for some job and find themselves lacking in ability.

These questions are designed to test your knowledge of this lesson. Read them over first to see if you can answer them. If you feel confident that you can, then write out your answers, numbering them to correspond to the questions. If you are not confident that you can answer the questions, re-study the lesson one or more times before writing out your answers. Be sure to answer every question, for if you fail to answer a question, it will reduce your grade on this lesson. When all questions have been answered, mail them to us for grading.

QUESTIONS

No. 1. What basic electrical network symbol is used as the equivalent of a transmission line for purpose of analysis?

No. 2. How many values of output impedance for a T section may be found that will make the input impedance have the same value as the output?

No. 3. What is the relation of the currents entering successive sections in the circuit of Fig. 9?

No. 4. Define the electrical susceptance of a circuit.

No. 5. The value obtained from the square root of the product of the open and closed circuit impedance of a transmission line relates to an important characteristic of the line. What is the name of this value?

No. 6. Is the characteristic impedance of a transmission line at audio frequencies of greater importance for a long line (several miles) or a short line (less than 50 feet)?

No. 7. If a transmission line is to have no reflection, how must its load impedance compare with its characteristic impedance?

No. 8. Why are steps of attenuation measured in db or ratio values rather than in volts, millivolts, microvolts, etc?

No. 9. What is the difference between an equalizer and a pad as used in transmission lines?

No. 10. What are the two general classifications of filters used in transmission lines?